Monitoring quantum systems

speed limits and symmetry breaking

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(La Plata)
Dynamics of open systems

- Quantum simulators
- Cryptography
- Quantum mechanical advantages
- Quantum technologies
- Q computer

Big problem: Quantum behavior is fragile. Evolution is unitary only under extremely controlled laboratory conditions. Quantum traits disappear, coherence, entanglement, non-classical correlations, effectively classical.
Dynamics of open systems

\[ \frac{d\rho_t}{dt} = -i[H, \rho_t] \]

We’d like instead, forced to deal with non-unitary dynamics

\[ \frac{d\rho_t}{dt} = -i[H, \rho_t] + D[\rho_t] \equiv L[\rho_t] \]
\[ = -i[H, \rho_t] - \kappa[A, [A, \rho_t]] \]

but, another situation can lead to identical dynamics: system being monitored by an observer

Environmental decoherence...
fluctuating Hamiltonians, uncontrolled sources of noise, uncertainties in the model...
Monitored quantum systems

System on which observable “A” is consecutively measured

Infinitesimally weak measurements – continuous quantum measurement
Monitored quantum systems

Weak measurement

\[ A = \sigma_z \]

\[
\begin{align*}
    (a|\uparrow\rangle + b|\downarrow\rangle) \left| \text{ready} \right\rangle & \quad \xrightarrow{t} \quad (a|\uparrow\rangle |\downarrow\rangle + b|\downarrow\rangle |\uparrow\rangle)
\end{align*}
\]

Measurement does not fully discriminate

Observing outcome “r” gives information of post-measurement state

Conditioning on “r” can be modeled by

\[
\rho \rightarrow \frac{\mathcal{M}_r \rho \mathcal{M}_r^+}{\text{Tr}[\mathcal{M}_r \rho \mathcal{M}_r^+]} \quad \mathcal{M}_r = \sum_{m=\pm 1} \frac{4\kappa dt}{\pi}^{1/4} e^{-2\kappa dt (r-m)^2} |m\rangle \langle m|
\]

“Quantum Measurement Theory and its Applications” K. Jacobs
“Quantum Measurement and Control” Wiseman and Milburn
Monitored quantum systems

Equivalent master equation:

\[ d\rho_t^C = -i[H, \rho_t^C]dt + D[\rho_t^C]dt + I[\rho_t^C]dW_t \]

\[ = -i[H, \rho_t^C]dt - \kappa [A, [A, \rho_t^C]] dt - \sqrt{2\kappa} \{A, \rho_t^C\} - 2 \text{ Tr}[A \rho_t^C \rho_t^C] dW_t \]

\[ \rho_t^C \rightarrow \text{conditioned state} \quad dW_t \rightarrow \text{white noise:} \quad \langle dW_t \rangle = 0; \quad \langle dW_{t_1} dW_{t_2} \rangle = \delta_{(t_1, t_2)} dt \]

\[ \kappa \rightarrow \text{measurement strength – backaction and timescale of information acquisition} \]

measurement output: observable masked by noise

\[ r_t dt = \text{Tr}(\rho_t A) dt + \frac{1}{\sqrt{8\kappa}} dW_t \]

output tracks expectation value of observable
Monitored quantum systems

measurement output: observable cloaked by noise

$$r_t dt = \text{Tr}(\rho_t^c A) dt + \frac{1}{\sqrt{8\kappa}} dW_t$$

$$\left( \rho \rightarrow \frac{M_t \rho M_t^\dagger}{\text{Tr} [M_t \rho M_t^\dagger]} \right)$$

Observing single quantum trajectories of a superconducting quantum bit

K. W. Murch\textsuperscript{1,2}, S. J. Weber\textsuperscript{1}, C. Macklin\textsuperscript{1} & I. Siddiqi\textsuperscript{1}

Weber et. al.; Comptes Rendus Physique 2016
Monitored quantum systems

measurement output: observable cloaked by noise

\[ r_t dt = \text{Tr}(\rho_t^C A) dt + \frac{1}{\sqrt{8\kappa}} dW_t \]

\[ \rho \rightarrow \frac{M_r \rho M_r^\dagger}{\text{Tr}[M_r \rho M_r^\dagger]} \]

filtering

path reconstruction

Garcia-Pintos and Justin Dressel; PRA 2017
Monitored quantum systems

Observer with access to outcomes describes system by

\[ d\rho_t^C = -i[H, \rho_t^C]dt + D[\rho_t^C]dt + I[\rho_t^C]dW_t = L[\rho_t^C]dt + I[\rho_t^C]dW_t \]

In contrast, agent without access describes system by \( \rho_t \equiv \langle \rho_t^C \rangle \), averaging out the unknown random results

\[ d\rho_t = \langle d\rho_t^C \rangle = -i[H, \langle \rho_t^C \rangle]dt + D[\langle \rho_t^C \rangle]dt + \langle I[\rho_t^C]dW_t \rangle = -i[H, \rho_t]dt + D[\rho_t]dt \approx 0 \]

\[ = L[\rho_t]dt \]

identical to open system dynamics!
Monitored quantum systems

\[ d\rho_t = \langle d\rho_t^C \rangle = -i[H, \rho_t]dt + D[\rho_t]dt \]
\[ = L[\rho_t]dt \]

\[ d\rho_t^C = -i[H, \rho_t^C]dt + D[\rho_t^C]dt + I[\rho_t^C]dW_t \]
\[ = L[\rho_t^C]dt + I[\rho_t^C]dW_t \]

Our motivation: study difference between conclusions from both descriptions
Setting 1

Speed of evolution and Quantum Speed Limits

joint with Luis Pedro Garcia-Pintos,

Quantum Speed Limits

Beautiful history
Passage time: Minimum time required for a state to reach an orthogonal state

Landau
Krylov

1945 Mandelstam and Tamm “MT”
1967 Fleming
1990 Anandan, Aharonov
1992 Vaidman, Ulhman
1993 Uffnik

1998 Margolus & Levitin “ML”
2000 Lloyd
2003 Giovannetti, Lloyd, Maccone: MT & ML unified
2003 Bender: no bounds in PT-symmetric QM
2009 Levitin, Toffoli

2013 2013 Bound for open (as well as unitary) system dynamics!
Speed of evolution

Limits to the speed of evolution

Mandelstam Tamm

\[
\left| \frac{d \text{Tr}(A\rho_t)}{dt} \right| \leq \Delta_{\rho_0} H \Delta_{\rho_0} A
\]

Margolus Levitin

\[
\tau_T \geq \frac{1}{2 \text{Tr}(\rho_0 H)}
\]

Fundamental limits for systems evolving unitarily

Extensions to open systems governed by Lindbladian dynamics

\[
\frac{d \rho_t}{dt} = -i[H, \rho_t] + D[\rho_t]
\]

Mandelstam and Tamm, J. Phys. (USSR) 1945
Aharonov and Bohm, Phys. Rev. 1961
Margolus and Levitin, Phys. D 1998

Taddei, Escher, Davidovich, de Matos Filho; PRL 2013
del Campo, Egusquiza, Plenio, S. F. Huelga; PRL 2013
Deffner and Lutz; PRL 2013
Limits to the speed of evolution

consider Fidelity
quantify deviation
from (pure) initial state

\[ F(t) = \text{Tr}[\rho_0 \rho_t] \]

Fidelity change after \( \tau \)

\[ \Delta F = \int_0^\tau \dot{F}(t) \, dt = \int_0^\tau \text{Tr}[\rho_0 \dot{\rho}_t] \, dt = -\tau \mathcal{V} \]

with the velocity

\[ \mathcal{V} \equiv -\frac{1}{\tau} \int_0^\tau \text{Tr}[\rho_0 \dot{\rho}_t] \, dt \]

\( d\rho_t = \langle d\rho_t^c \rangle = -i[H, \rho_t]dt + \mathcal{D}[\rho_t]dt \)

\( d\rho_t^c = -i[H, \rho_t^c]dt + \mathcal{D}[\rho_t^c]dt + \mathcal{I}[\rho_t^c]dW_t \)
Limits to the speed of evolution

\[ d\rho_t = \langle d\rho^c_t \rangle = L[\rho_t]dt \]

\textbf{velocity} \quad \nu \equiv -\frac{1}{\tau} \int_0^\tau \text{Tr}[\rho_0 \langle d\rho^c_t \rangle]

\textbf{Ignorant agent thus expects} \quad \nu \leq \nu_{QSL}

\nu_{QSL} = \frac{1}{\tau} \int_0^\tau \|L(\rho_t)\| dt

\textbf{traditional bound on speed studied in literature}

\[ d\rho^c_t = L[\rho^c_t]dt + L[\rho^c_t]dW_t \]

\textbf{Agent with outcomes knows better:}

\[ \nu_c \equiv -\frac{1}{\tau} \int_0^\tau \text{Tr}[\rho_0 L[\rho^c_t]]dt - \frac{1}{\tau} \int_0^\tau \text{Tr}[\rho_0 J[\rho^c_t]]dW_t \]

Garcia-Pintos and del Campo, arXiv 2018
Limits to the speed of evolution

Agent with access to outcomes finds

\[ \langle V_c \rangle = V \quad \langle V_c^2 \rangle \neq 0 \]

velocity is a random variable!

Example on qubit, monitoring of \( \sigma_z \)

\[ H = \frac{\omega}{2} \sigma_y \]

Garcia-Pintos and del Campo, arXiv 2018
Speed limits

Distribution of travel times to a target Fidelity

Garcia-Pintos and del Campo, arXiv 2018
Speed limits

Traditional derivations of speed limits had focused on

\[
\frac{d \rho_t}{dt} = -i[H, \rho_t] + D[\rho_t]
\]

\[
= -i[H, \rho_t] - \kappa [A, [A, \rho_t]]
\]

Extended to monitored systems, dynamics non-linear in state

\[
d \rho_t^C = -i[H, \rho_t^C]dt + D[\rho_t^C]dt + I[\rho_t^C]dW_t
\]

\[
= -i[H, \rho_t^C]dt - \kappa [A, [A, \rho_t^C]] dt - \sqrt{2\kappa}(\{A, \rho_t^C\} - 2 \text{Tr}[A \rho_t^C] \rho_t^C) dW_t
\]

Velocity becomes stochastic, with trajectories traveling faster than what an agent ignorant of measurement outcomes would expect

Garcia-Pintos and del Campo, arXiv 2018
Monitoring of a many-body system: symmetry breaking

joint with Luis Pedro Garcia-Pintos and Diego Tielas
Spontaneous symmetry breaking: process by which one state is singled out, out of a set indistinguishable by the dynamics.
Symmetry breaking

Usual ways to explain it:

- Tiny perturbation to Hamiltonian
- Tiny perturbation to state

We consider: quantum monitoring as cause of symmetry breaking

Garcia-Pintos, Tielas, del Campo, arXiv 2018
Symmetry breaking

spin chain, initially

\[ |\Psi(0)\rangle = \bigotimes_{j=1}^{N} |\rightarrow\rangle_j \]

quenched to Hamiltonian

\[ H = \Delta \sum_j \sigma_j^z \sigma_{j+1}^z \]

monitoring of individual spins

\[ d\rho_t^c = \langle d\rho_t^c \rangle = L[\rho_t]dt \]

\[ d\rho_t^c = L[\rho_t^c]dt + \sum_j I_j[\rho_t^c]dW_t^j \]
Symmetry breaking

evolution expected by ignorant agent does not distinguish states!
monitoring term singles out a direction

\[ d\rho_t^C = \langle d\rho_t^C \rangle = L[\rho_t]dt \]

\[ L[\rho_t] = -i[H, \rho_t] - \kappa \sum_j \left[ \sigma_j^z, [\sigma_j^z, \rho_t] \right] \]

\( \langle \rho_t^C \rangle \)

\( \text{dephasing operators} \ [H, \theta_j^z] = 0 \)

\( \text{do not break symmetry} \)

\[ d\rho_t^C = L[\rho_t^C]dt + \sum_j I_j[\rho_t^C] dW_t^j \]

\[ I_j[\rho_t^C] = \sqrt{2\kappa} (\{\sigma_j^z, \rho_t^C\} - 2 Tr[\sigma_j^z \rho_t^C] \rho_t^C) \]

\[ d\text{Tr}[\sigma_j^z \rho_t^C] = -\sqrt{8\kappa} \text{var}(\sigma_j^z, \rho_t^C) dW_t^j \]

\( \text{fixed states} |\uparrow\rangle \text{ or } |\downarrow\rangle ! \)
Symmetry breaking

Garcia-Pintos, Tielas, del Campo, arXiv 2018
Symmetry breaking – effect of measurements

Coarse grained measurements

Garcia-Pintos, Tielas, del Campo, arXiv 2018
Symmetry breaking

Evolution expected by ignorant agent does not distinguish states!

Monitoring breaks symmetry in each realization

\[ \langle \rho_t^c \rangle \]

Monitoring agent can influence properties of symmetry-broken state

Garcia-Pintos, Tielas, del Campo, arXiv 2018
Outlook: Phase transitions in finite time: Defect suppression

SO FAR: ISOLATED SYSTEMS, NO MONITORING

What is known: Kibble-Zurek mechanism (KZM) predicts mean density of topological defects

News beyond KZM:
Full counting statistics of topological defects
“Quantum speed limits under continuous quantum measurements”,
Luis Pedro García-Pintos and Adolfo del Campo,

“Spontaneous symmetry breaking induced by quantum monitoring”,
Luis Pedro García-Pintos, Diego Tielas, and Adolfo del Campo,

“Universal Statistics of Topological Defects Formed in a Quantum Phase Transition”