Monitoring quantum systems

speed limits and symmetry breaking

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DIPC

Engineering Quantum Open Systems

Workshop EQOS2019

February 12th, 2019

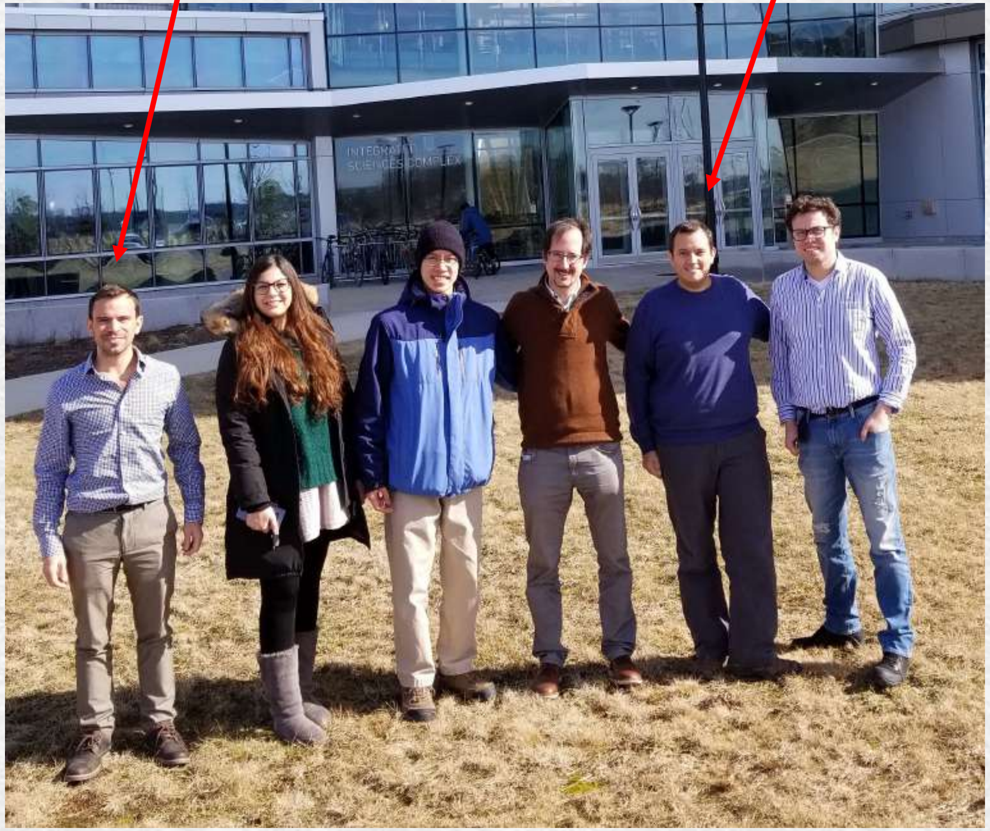
Quantum Science & Technology group @ UMass Boston

Quantum Science & Technology group @ UMass Boston

Luis Pedro García-Pintos

Diego Tielas

(La Plata)



Quantum Science & Technology group @ DIPC & UPV-EHU

Dynamics of open systems

trade of exploiting quantum mechanical advantages

quantum simulators

cryptography

quantum technologies

Q computer

big problem: quantum behavior is fragile

evolution is unitary only under extremely controlled laboratory conditions

quantum traits disappear

~~coherence~~
~~entanglement~~
~~non-classical correlations~~

effectively classical

Dynamics of open systems

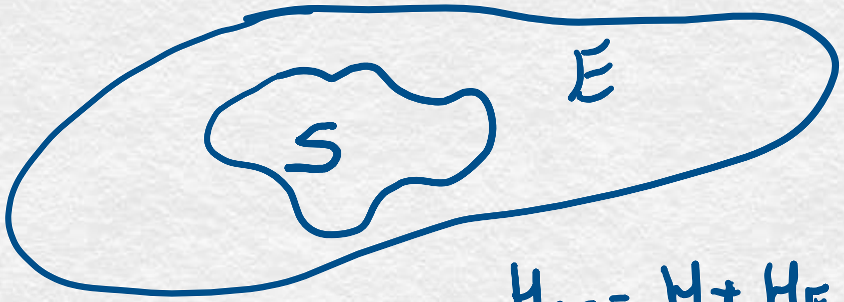
We'd like

$$\frac{d\rho_t}{dt} = -i[H,\rho_t]$$

instead, forced to deal with non-unitary dynamics

$$\frac{d\rho_t}{dt} = -i[H,\rho_t] + \mathcal{D}[\rho_t] \equiv L[\rho_t]$$

$$= -i[H,\rho_t] - \kappa[A,[A,\rho_t]]$$



$$H_{TOT} = H + H_E + H_{int}$$

$$\rho_t = Tr_E(\rho_{TOT}(t))$$

Environmental decoherence

…

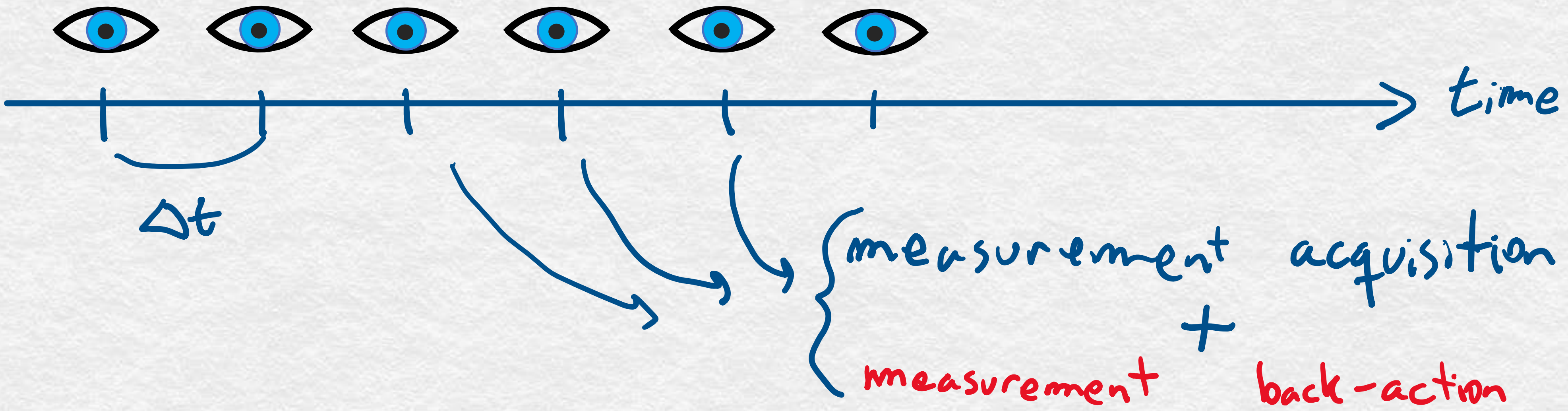
fluctuating Hamiltonians,
uncontrolled sources of noise,
uncertainties in the model…

but, another situation can lead to identical dynamics:

system being monitored by an observer

Monitored quantum systems

System on which observable "**A**" is consecutively measured



ρ_{t_1} ρ_{t_2} ρ_{t_3} . . . .

Infinitesimally weak measurements – continuous quantum measurement

Monitored quantum systems

Weak measurement

$A = \sigma_z$

$$\underbrace{(a|\uparrow\rangle + b|\downarrow\rangle)}_{\text{measured system}}\underbrace{|\text{ready}\rangle}_{\text{apparatus}} \xrightarrow{t} (a|\uparrow\rangle|\psi_\uparrow\rangle + b|\downarrow\rangle|\psi_\downarrow\rangle)$$

measurement does not fully discriminate

$|\langle\psi_\uparrow|\psi_\downarrow\rangle| \approx 1$

measurement readout

Observing outcome "**r**" gives information of post-measurement state

Conditioning on "r" can be modeled by

$$\rho \longrightarrow \frac{\mathcal{M}_r \rho \mathcal{M}_r^\dagger}{Tr[\mathcal{M}_r \rho \mathcal{M}_r^\dagger]}$$

$$\mathcal{M}_r = \sum_{m=\pm 1} \left(\frac{4\kappa dt}{\pi}\right)^{1/4} e^{-2\kappa dt (r-m)^2} |m\rangle\langle m|$$

Gaussian measurements

"Quantum Measurement Theory and its Applications" K. Jacobs
"Quantum Measurement and Control" Wiseman and Milburn

Monitored quantum systems

Equivalent master equation:

information acquisition { purifies / back-action / non-linear

$$d\rho_t^{\mathcal{C}} = -i[H,\rho_t^{\mathcal{C}}]dt + \mathcal{D}[\rho_t^{\mathcal{C}}]dt + I[\rho_t^{\mathcal{C}}]dW_t$$

$$= -i[H,\rho_t^{\mathcal{C}}]dt - \kappa\left[A,[A,\rho_t^{\mathcal{C}}]\right]dt - \sqrt{2\kappa}(\{A,\rho_t^{\mathcal{C}}\} - 2\,\mathrm{Tr}[A\rho_t^{\mathcal{C}}]\rho_t^{\mathcal{C}})dW_t$$

$\rho_t^{\mathcal{C}} \rightarrow$ *conditioned* state

$dW_t \rightarrow$ white noise: $\langle dW_t\rangle = 0;$ $\langle dW_{t_1}dW_{t_2}\rangle = \delta_{\{t_1,t_2\}}dt$

$\kappa \rightarrow$ measurement strength – backaction and timescale of information acquisition

measurement output: observable masked by noise

$$r_t dt = Tr(\rho_t A)dt + \frac{1}{\sqrt{8\kappa}}dW_t$$

→ output tracks expectation value of observable

Monitored quantum systems

measurement output:
observable cloaked by noise

$$r_t dt = \mathrm{Tr}(\rho_t^{\mathcal{C}} A)dt + \frac{1}{\sqrt{8\kappa}} dW_t$$

$$\left(\rho \longrightarrow \frac{\mathcal{M}_r \rho \mathcal{M}_r^\dagger}{Tr\left[\mathcal{M}_r \rho \mathcal{M}_r^\dagger\right]}\right)$$

LETTER

doi:10.1038/nature12539

Observing single quantum trajectories of a superconducting quantum bit

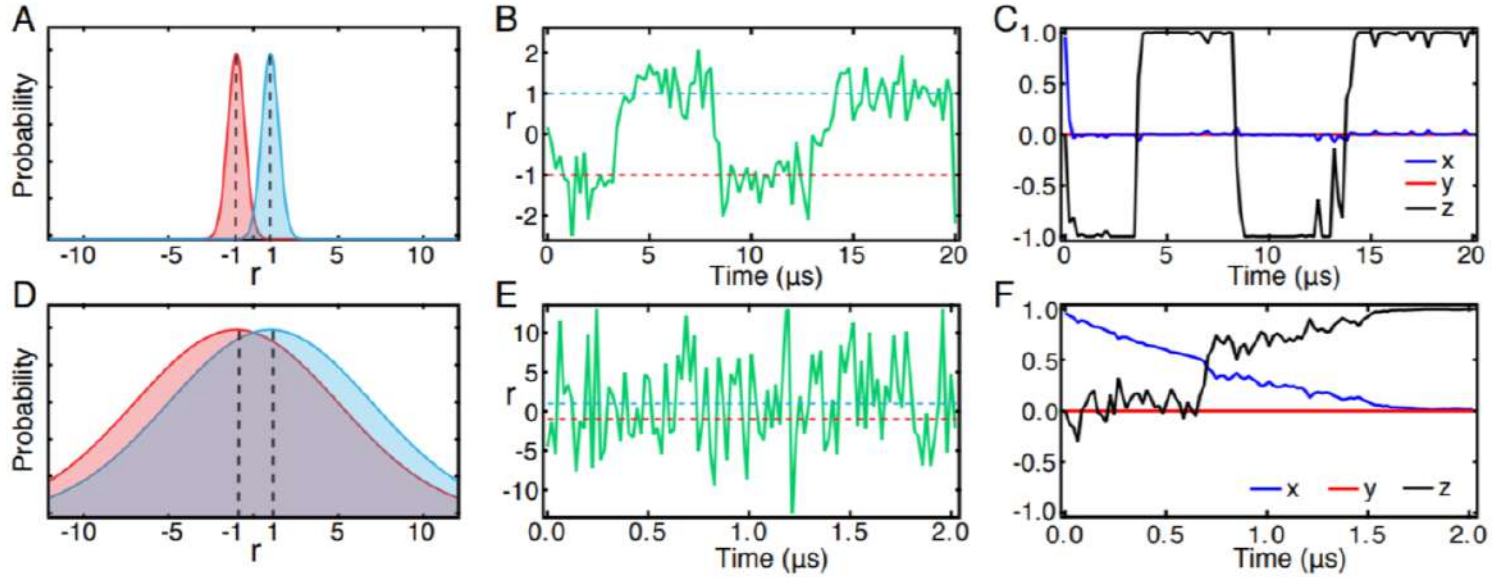
K. W. Murch[1,2], S. J. Weber[1], C. Macklin[1] & I. Siddiqi[1]

QUANTUM PHYSICS

Watching the wavefunctio

The continuous random path of a supercond been tracked as the state changes during me possibility of steering quantum systems int

ANDREW N. JORDAN



0.0 0.4 0.8 1.2 1.6
Time (μs)

Figure 3 | Quantum trajectories. a, b, Individual mea

Murch, Weber, Macklin, and Siddiqi; Nature 2013
Weber et. al.; Comptes Rendus Physique 2016

Monitored quantum systems

measurement output:
observable cloaked by noise

$$r_t dt = \mathrm{Tr}(\rho_t^{\mathcal{C}} A)dt + \frac{1}{\sqrt{8\kappa}} dW_t$$

$$\left(\rho \longrightarrow \frac{\mathcal{M}_r \rho \mathcal{M}_r^\dagger}{Tr\left[\mathcal{M}_r \rho \mathcal{M}_r^\dagger\right]}\right)$$

filtering

path reconstruction

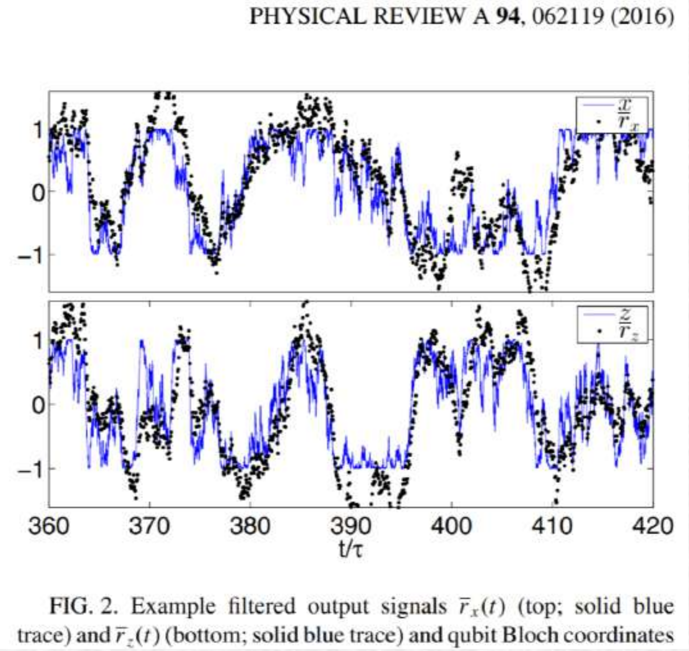
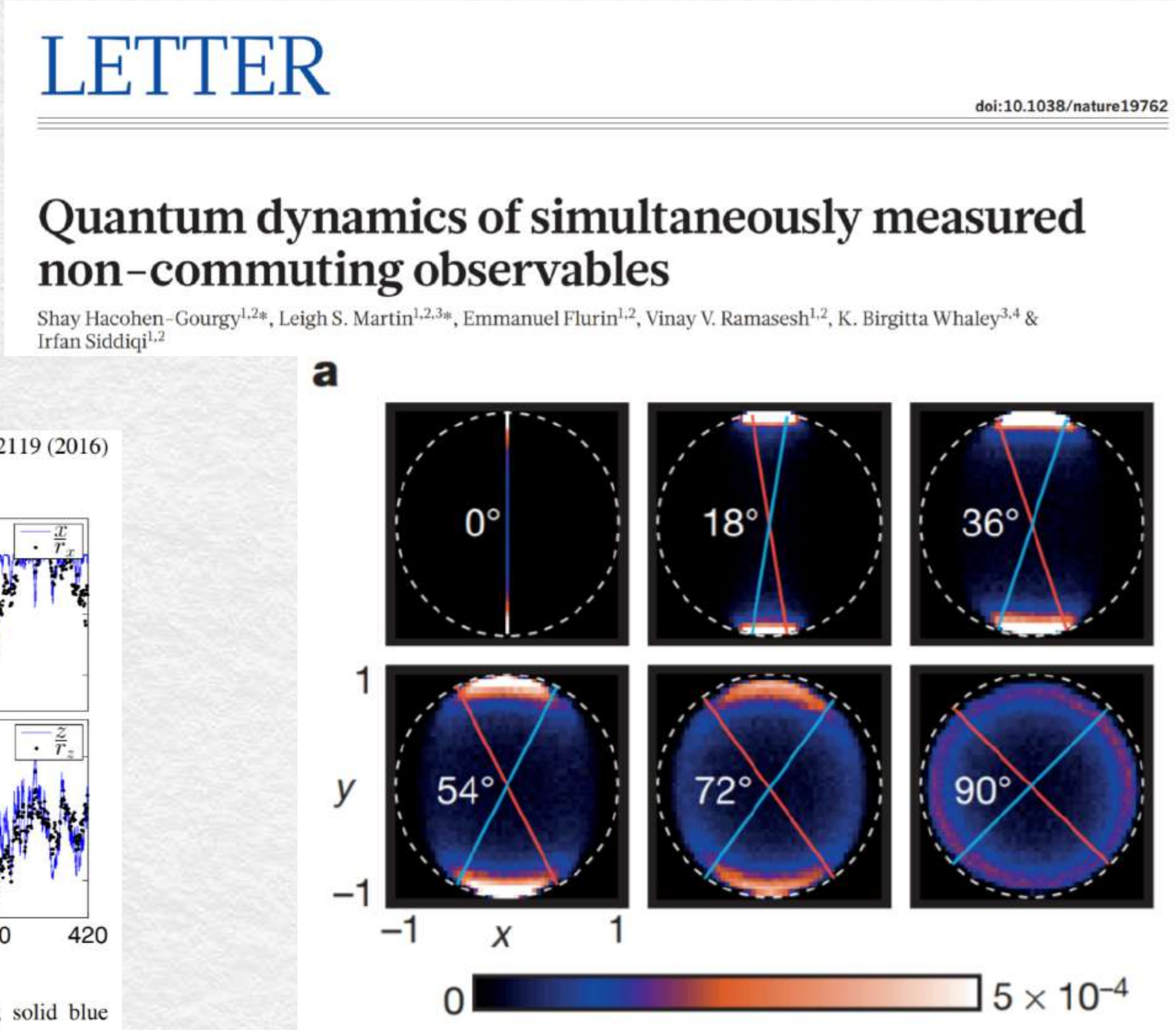

PHYSICAL REVIEW A **94**, 062119 (2016)

FIG. 2. Example filtered output signals $\overline{r}_x(t)$ (top; solid blue trace) and $\overline{r}_z(t)$ (bottom; solid blue trace) and qubit Bloch coordinates


LETTER

doi:10.1038/nature19762

Quantum dynamics of simultaneously measured non-commuting observables

Shay Hacohen-Gourgy[1,2*], Leigh S. Martin[1,2,3*], Emmanuel Flurin[1,2], Vinay V. Ramasesh[1,2], K. Birgitta Whaley[3,4] & Irfan Siddiqi[1,2]

Hacohen-Gourgy, Martin, Flurin, Ramasesh, Whaley, and Siddiqi; Nature 2016
Garcia-Pintos and Justin Dressel; PRA 2017

Monitored quantum systems

Observer with access to outcomes describes system by

$$d\rho_t^c = -i[H, \rho_t^c]dt + \mathcal{D}[\rho_t^c]dt + I[\rho_t^c]dW_t = L[\rho_t^c]dt + I[\rho_t^c]dW_t$$

In contrast, agent without access describes system by $\rho_t \equiv \langle \rho_t^c \rangle$, averaging out the unknown random results

$$d\rho_t = \langle d\rho_t^c \rangle = -i[H, \langle \rho_t^c \rangle]dt + \mathcal{D}[\langle \rho_t^c \rangle]dt + \langle I[\rho_t^c]dW_t \rangle$$

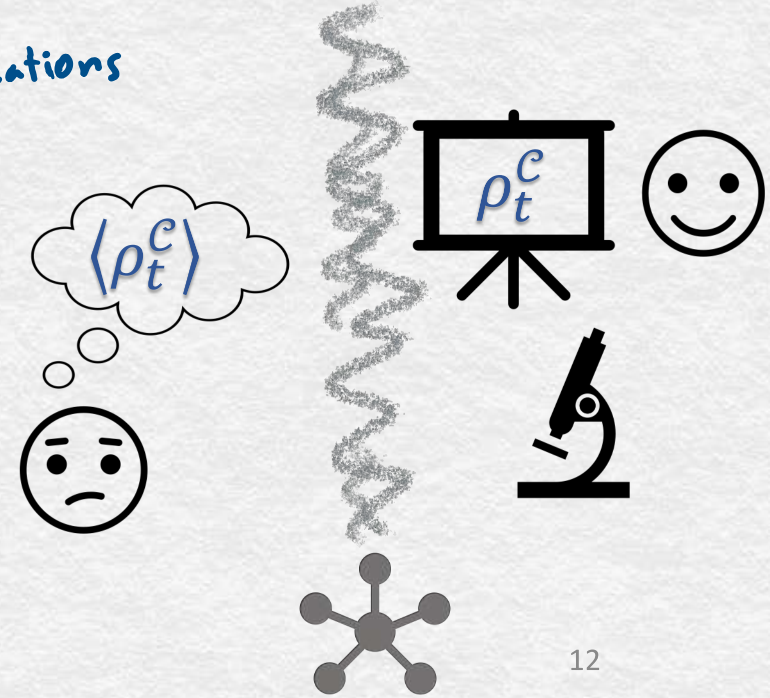
$$= -i[H, \rho_t]dt + \mathcal{D}[\rho_t]dt$$

$$= L[\rho_t]dt$$

→ average over realizations

= 0

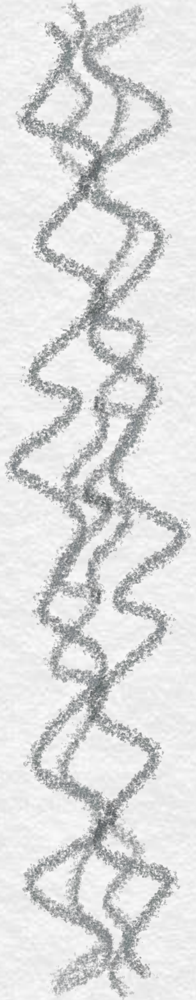
identical to open system dynamics!



Monitored quantum systems

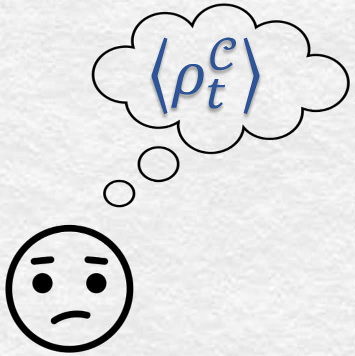
$$d\rho_t = \langle d\rho_t^{\mathcal{C}} \rangle = -i[H, \rho_t]dt + \mathcal{D}[\rho_t]dt$$

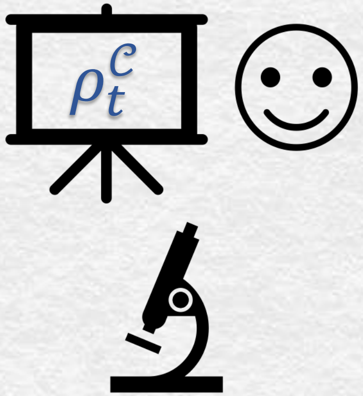
$$= L[\rho_t]dt$$



$$d\rho_t^{\mathcal{C}} = -i[H, \rho_t^{\mathcal{C}}]dt + \mathcal{D}[\rho_t^{\mathcal{C}}]dt + I[\rho_t^{\mathcal{C}}]dW_t$$

$$= L[\rho_t^{\mathcal{C}}]dt + I[\rho_t^{\mathcal{C}}]dW_t$$





Our motivation: study difference between conclusions from both descriptions

Setting 1

Speed of evolution and Quantum Speed Limits

joint with Luis Pedro Garcia-Pintos,
arXiv: 1804.01600 (2018)

Quantum Speed Limits

Courtesy of Guy Chenu

Quantum Speed Limits

Beautiful history

Passage time: Minimum time required for a state to reach an orthogonal state

Landau 

Krylov

1945 Mandelstam and Tamm "MT" 

1967 Fleming

1990 Anandan, Aharonov

1992 Vaidman, Ulhman

1993 Uffnik

1998 Margolus & Levitin "ML" 

2000 Lloyd

2003 Giovannetti, Lloyd, Maccone: MT & ML unified

2003 Bender: no bounds in PT-symmetric QM

2009 Levitin,Toffoli

2013 2013 Bound for open (as well as unitary) system dynamics!

Speed of evolution

Limits to the speed of evolution

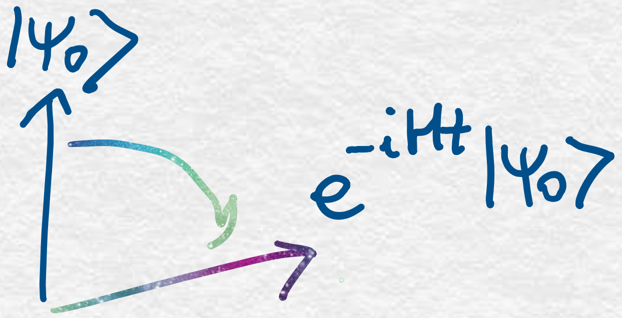
Mandelstam Tamm

$$\left|\frac{d\mathrm{Tr}(A\rho_t)}{dt}\right| \leq \Delta_{\rho_0} H \Delta_{\rho_0} A$$

$\langle \psi_0 | e^{iHt} A_0 e^{-iHt} | \psi_0 \rangle$

Margolus Levitin

$$\tau_T \geq \frac{1}{2\mathrm{Tr}(\rho_0 H)}$$



Fundamental limits for systems evolving unitarily

Extensions to open systems governed by Lindbladian dynamics

$$\frac{d\rho_t}{dt} = -i[H,\rho_t] + \mathcal{D}[\rho_t]$$

Mandelstam and Tamm, J. Phys. (USSR) 1945
Aharonov and Bohm, Phys. Rev. 1961
Margolus and Levitin, Phys. D 1998

Taddei, Escher, Davidovich, de Matos Filho; PRL 2013
del Campo, Egusquiza, Plenio, S. F. Huelga; PRL 2013
Deffner and Lutz; PRL 2013

Limits to the speed of evolution

consider Fidelity
quantify deviation
from (pure) initial state

$$F(t) = \mathrm{Tr}[\rho_0 \rho_t]$$

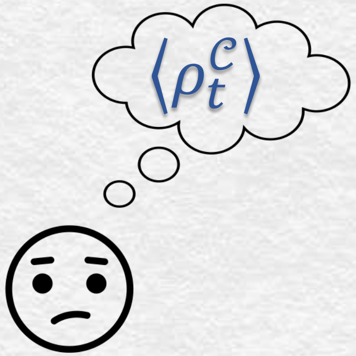
Fidelity change after τ

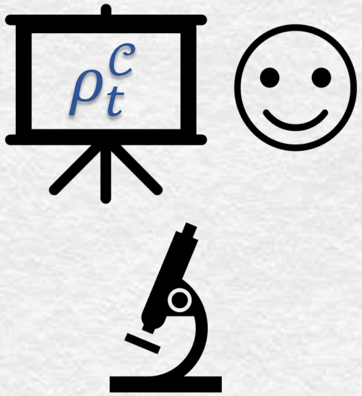
$$\Delta F = \int_0^{\tau} \dot{F}(t)\, dt = \int_0^{\tau} \mathrm{Tr}[\rho_0 \dot{\rho}_t] dt = -\tau \mathcal{V}$$

with the *velocity* $\mathcal{V} \equiv -\frac{1}{\tau}\int_0^{\tau} \mathrm{Tr}[\rho_0 \dot{\rho}_t]\, dt$

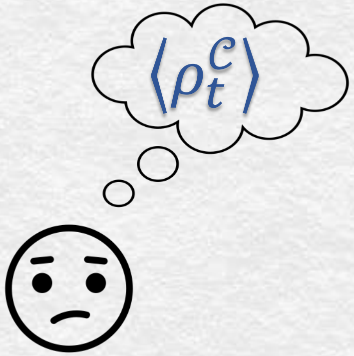
$$d\rho_t = \langle d\rho_t^{\mathcal{C}} \rangle = -i[H, \rho_t]dt + \mathcal{D}[\rho_t]dt$$

$$d\rho_t^{\mathcal{C}} = -i[H, \rho_t^{\mathcal{C}}]dt + \mathcal{D}[\rho_t^{\mathcal{C}}]dt + I[\rho_t^{\mathcal{C}}]dW_t$$





Limits to the speed of evolution



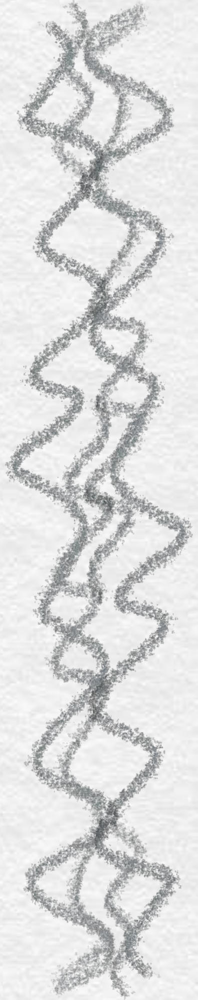
$$d\rho_t = \langle d\rho_t^{\mathcal{C}} \rangle = L[\rho_t]dt$$

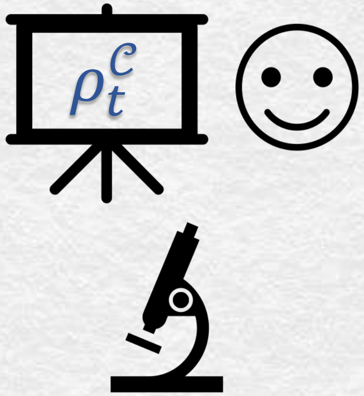
velocity $\mathcal{V} \equiv -\frac{1}{\tau}\int_0^\tau \mathrm{Tr}[\rho_0 \langle d\rho_t^{\mathcal{C}} \rangle]$

Ignorant agent thus expects $\mathcal{V} \leq \mathcal{V}_{QSL}$

$$\mathcal{V}_{QSL} = \frac{1}{\tau}\int_0^\tau \|\mathrm{L}(\rho_t)\| dt$$

traditional bound on speed studied in literature





$$d\rho_t^{\mathcal{C}} = L[\rho_t^{\mathcal{C}}]dt + I[\rho_t^{\mathcal{C}}]dW_t$$

Agent with outcomes knows better:

$$\mathcal{V}_{\mathcal{C}} \equiv -\frac{1}{\tau}\int_0^\tau \mathrm{Tr}[\rho_0 L[\rho_t^{\mathcal{C}}]]dt - \frac{1}{\tau}\int_0^\tau \mathrm{Tr}\left[\rho_0 I[\rho_t^{\mathcal{C}}]\right] dW_t$$

Garcia-Pintos and del Campo, arXiv 2018

Limits to the speed of evolution

Agent with access to outcomes finds

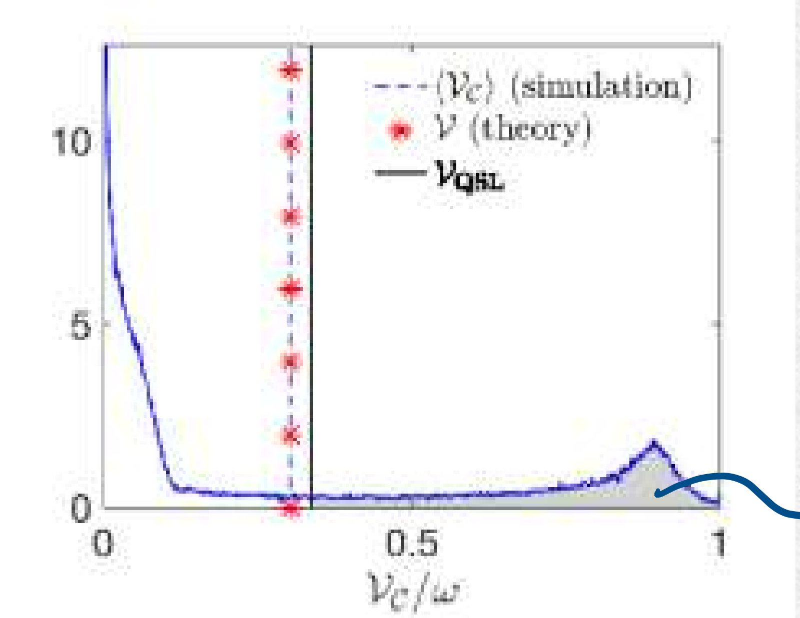
$$\langle \mathcal{V}_{\mathcal{C}} \rangle = \mathcal{V} \qquad \langle {\mathcal{V}_{\mathcal{C}}}^2 \rangle \neq 0$$

→ velocity is a random variable!

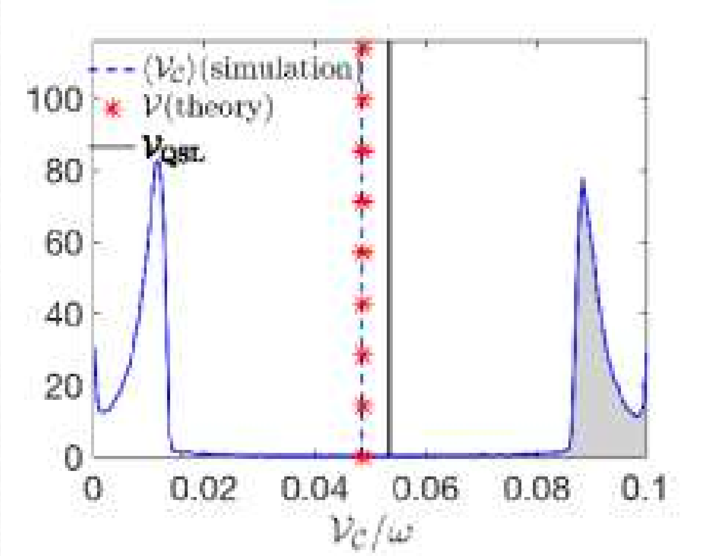
$$\langle {\mathcal{V}_{\mathcal{C}}}^2 \rangle = \left\langle \overline{\mathrm{Tr}\left(\rho_0 L[\rho_t^{\mathcal{C}}]\right)}^2 \right\rangle + \frac{2}{\tau}\left\langle \overline{\mathrm{Tr}\left(\rho_0 L[\rho_{t_1}^{\mathcal{C}}]\right)} \int_0^\tau \mathrm{Tr}\left(\rho_0 I[\rho_{t_2}^{\mathcal{C}}]\right) dW_{t_2} \right\rangle + \frac{1}{\tau^2}\left\langle \int_0^\tau \int_0^\tau \mathrm{Tr}\left(\rho_0 I[\rho_{t_1}^{\mathcal{C}}]\right) \mathrm{Tr}\left(\rho_0 I[\rho_{t_2}^{\mathcal{C}}]\right) dW_{t_1} dW_{t_2} \right\rangle . \qquad (15)$$

Example on qubit, monitoring of σ_z $\qquad H = \frac{\omega}{2}\sigma_y$

velocity distribution



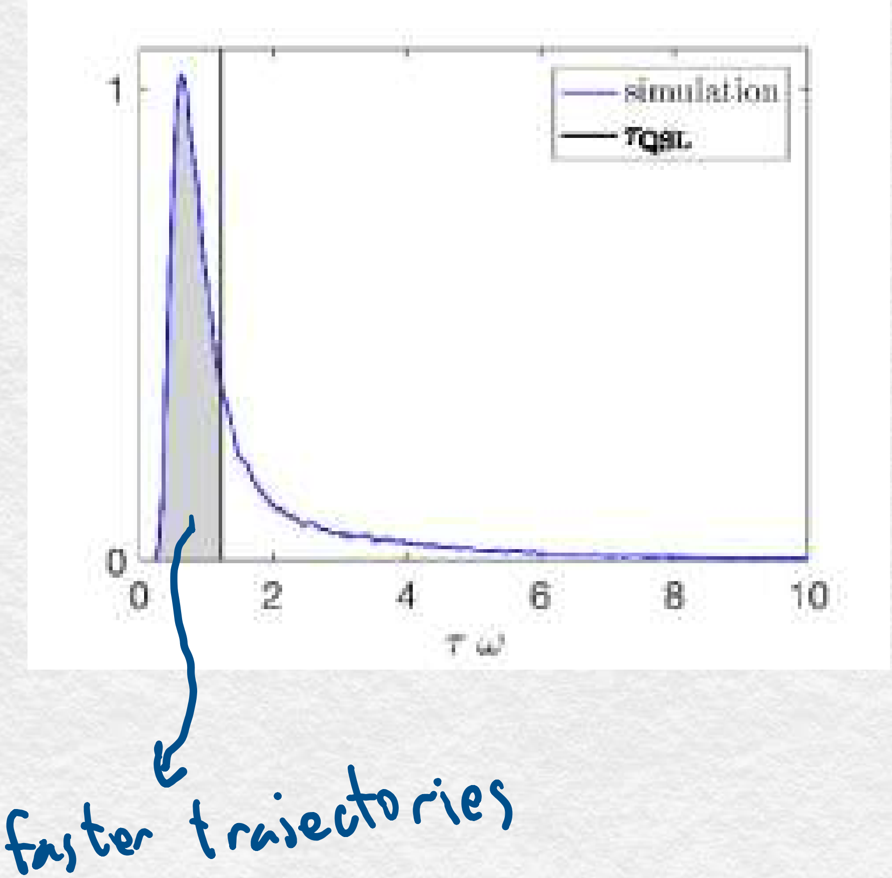
trajectories violating $\mathcal{V}_{QSL}$

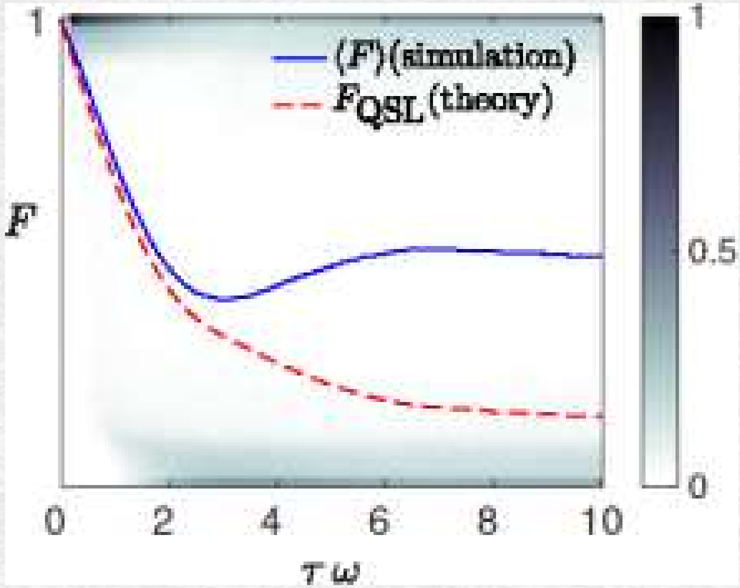


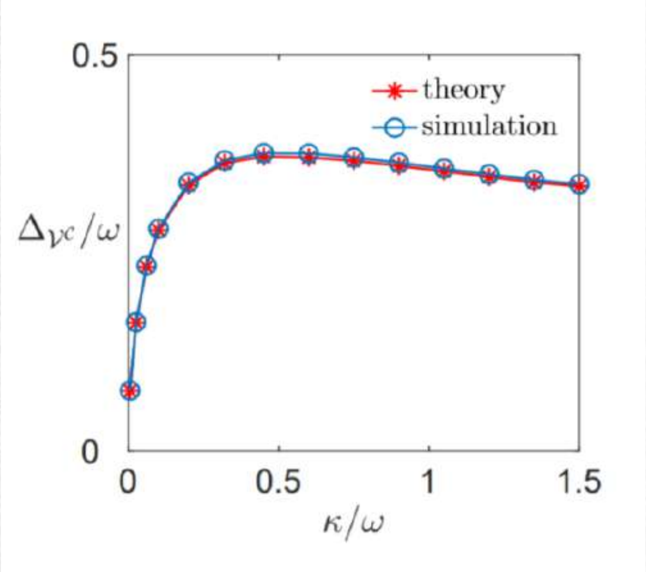
Garcia-Pintos and del Campo, arXiv 2018

Speed limits

Distribution of travel times to a target Fidelity







Garcia-Pintos and del Campo, arXiv 2018

Speed limits

Traditional derivations of speed limits had focused on

$$
\begin{aligned}
\frac{d\rho_t}{dt} &= -i[H,\rho_t] + \mathcal{D}[\rho_t] \\
&= -i[H,\rho_t] - \kappa\big[A,[A,\rho_t]\big]
\end{aligned}
$$

Extended to monitored systems, dynamics non-linear in state

$$
\begin{aligned}
d\rho_t^{\mathcal{C}} &= -i\big[H,\rho_t^{\mathcal{C}}\big]dt + \mathcal{D}\big[\rho_t^{\mathcal{C}}\big]dt + I\big[\rho_t^{\mathcal{C}}\big]dW_t \\
&= -i\big[H,\rho_t^{\mathcal{C}}\big]dt - \kappa\left[A,\big[A,\rho_t^{\mathcal{C}}\big]\right]dt - \sqrt{2\kappa}\big(\{A,\rho_t^{\mathcal{C}}\} - 2\,\mathrm{Tr}\big[A\rho_t^{\mathcal{C}}\big]\rho_t^{\mathcal{C}}\big)dW_t
\end{aligned}
$$

Velocity becomes stochastic, with trajectories traveling faster than what an agent ignorant of measurement outcomes would expect

Garcia-Pintos and del Campo, arXiv 2018

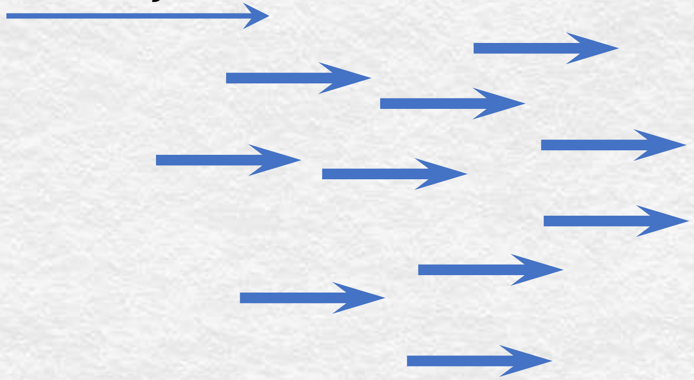
Setting 2

Monitoring of a many-body system: symmetry breaking

joint with Luis Pedro Garcia-Pintos and Diego Tielas
arXiv:1808.08343 (2018)

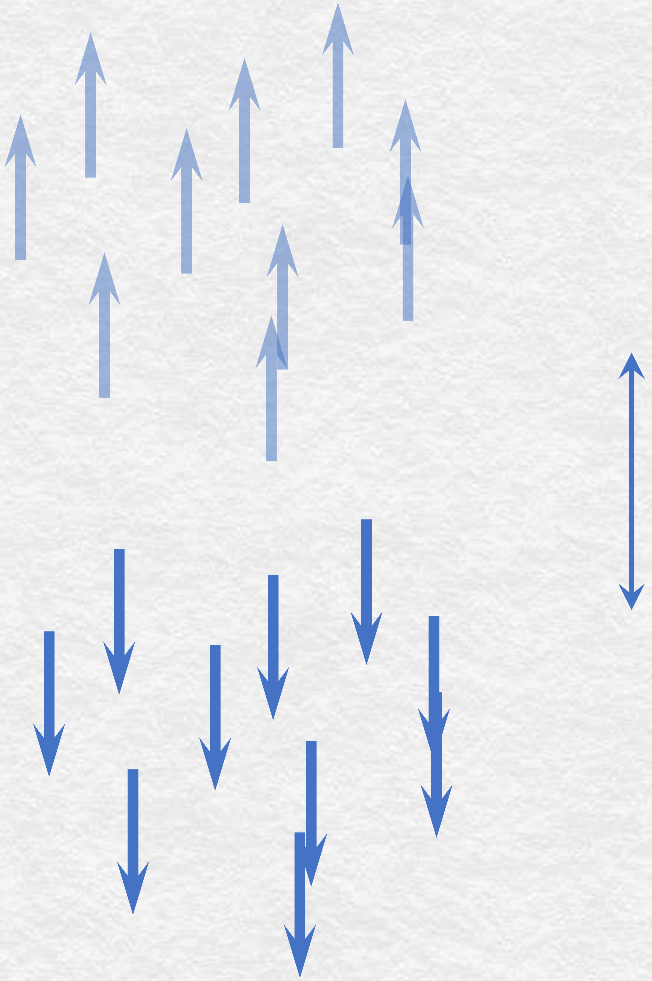
Symmetry breaking

$$H_0 \propto \sum_j \sigma_j^x$$



quench to Hamiltonian
with degenerate
ground state

$$H \propto \sum_j \sigma_j^z \sigma_{j+1}^z$$

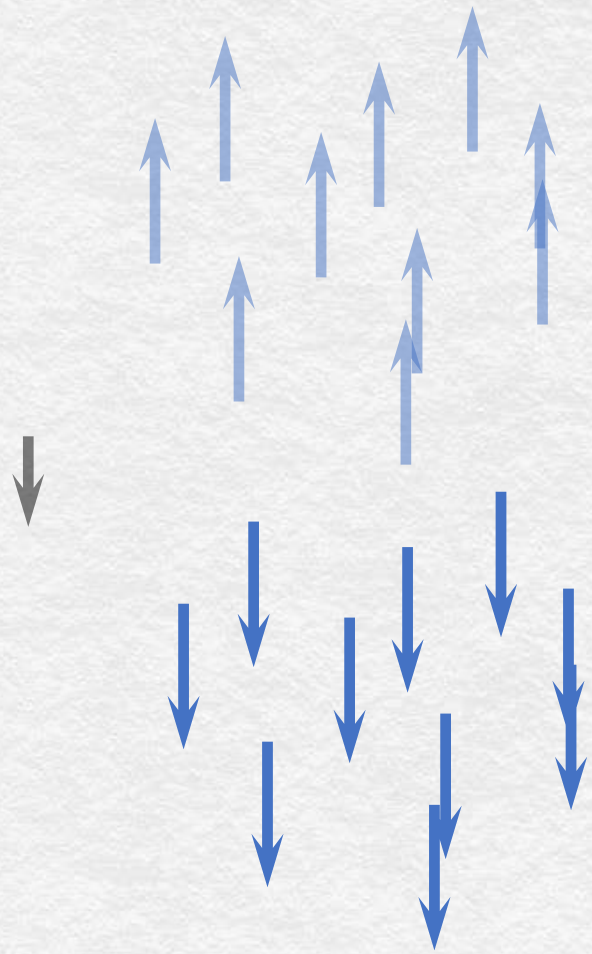


Spontaneous symmetry breaking:
process by which *one* state is singled out,
out of a set indistinguishable by the dynamics

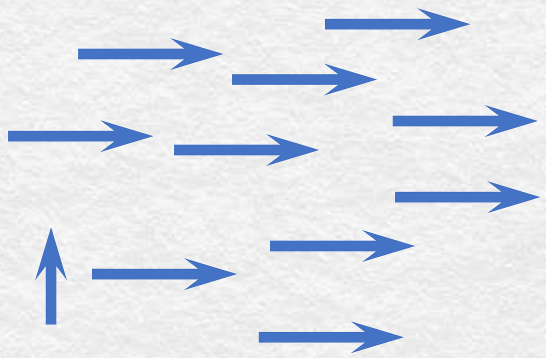
Symmetry breaking

Usual ways to explain it:

tiny perturbation to Hamiltonian



tiny perturbation to state



we consider: *quantum monitoring as cause of symmetry breaking*

Garcia-Pintos, Tielas, del Campo, arXiv 2018

Symmetry breaking

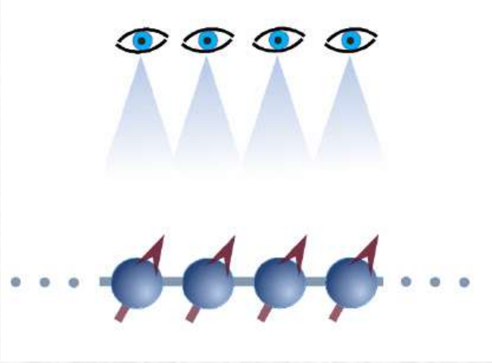
spin chain, initially

$$|\Psi(0)\rangle = \bigotimes_{j=1}^{N} |\rightarrow\rangle_j$$

quenched to Hamiltonian

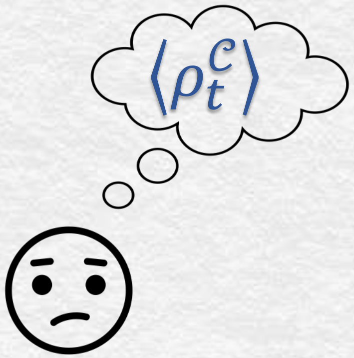
$$H = \Delta \sum_j \sigma_j^z \sigma_{j+1}^z$$

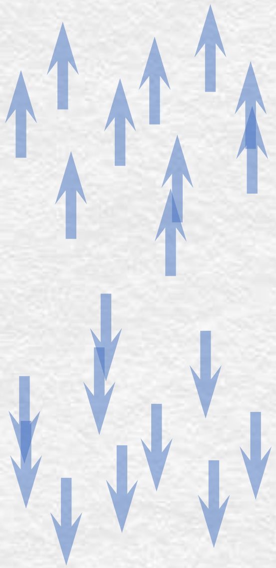
monitoring of individual spins

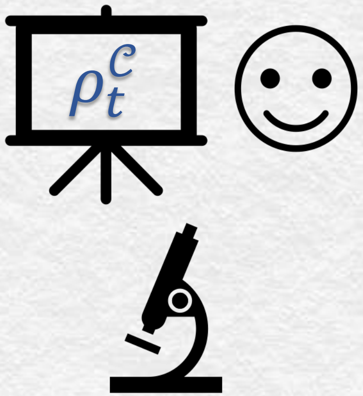


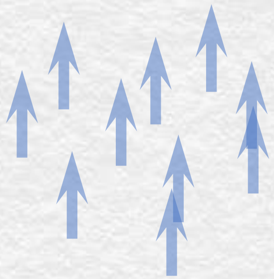
$$d\rho_t = \langle d\rho_t^c \rangle = L[\rho_t]dt$$

$$d\rho_t^c = L[\rho_t^c]dt + \sum_j I_j[\rho_t^c]\, dW_t^j$$



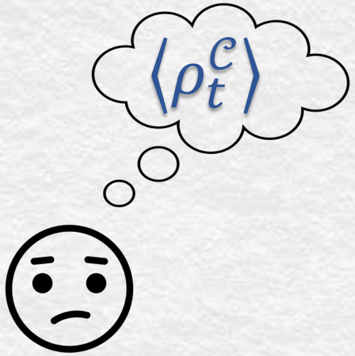


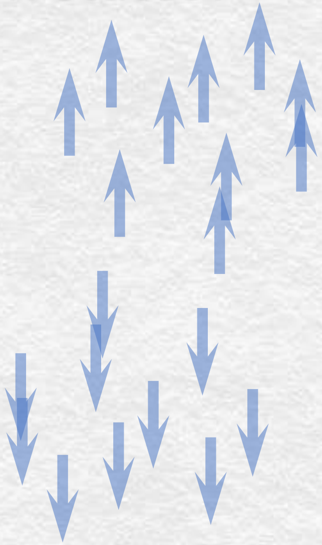




Symmetry breaking

evolution expected by ignorant agent does not distinguish states!



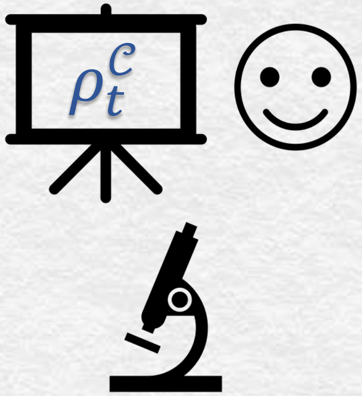


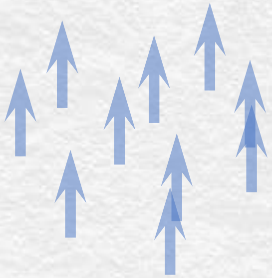
$$d\rho_t = \langle d\rho_t^{\mathcal{C}} \rangle = L[\rho_t]dt$$

$$L[\rho_t] = -i[H,\rho_t] - \kappa \sum_j \left[\sigma_j^z, [\sigma_j^z, \rho_t]\right]$$

↳ dephasing operators $[H, \sigma_j^z] = 0$ do not break symmetry

monitoring term singles out a direction





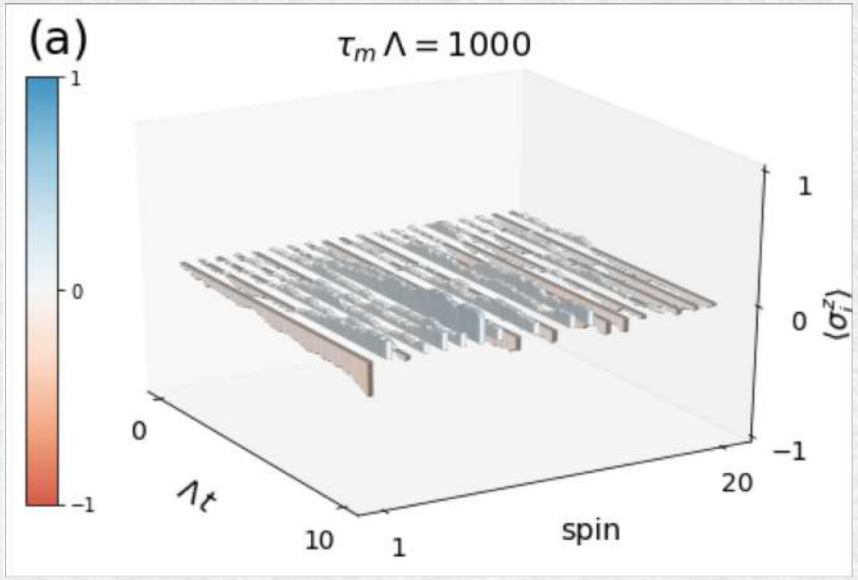
$$d\rho_t^{\mathcal{C}} = L[\rho_t^{\mathcal{C}}]dt + \sum_j I_j[\rho_t^{\mathcal{C}}]\, dW_t^j$$

$$I_j[\rho_t^{\mathcal{C}}] = \sqrt{2\kappa}\left(\{\sigma_j^z, \rho_t^{\mathcal{C}}\} - 2Tr[\sigma_j^z \rho_t^{\mathcal{C}}]\rho_t^{\mathcal{C}}\right)$$

$$d\mathrm{Tr}[\sigma_j^z \rho_t^{\mathcal{C}}] = -\sqrt{8\kappa}\,\mathrm{var}(\sigma_j^z, \rho_t^{\mathcal{C}})dW_t^j$$

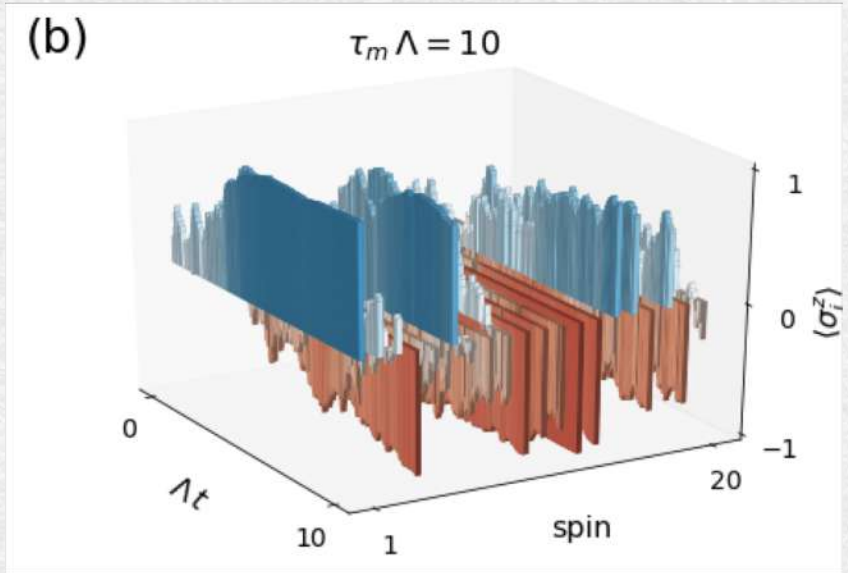
↳ fixed states $|\uparrow\rangle$ or $|\downarrow\rangle$!

Symmetry breaking

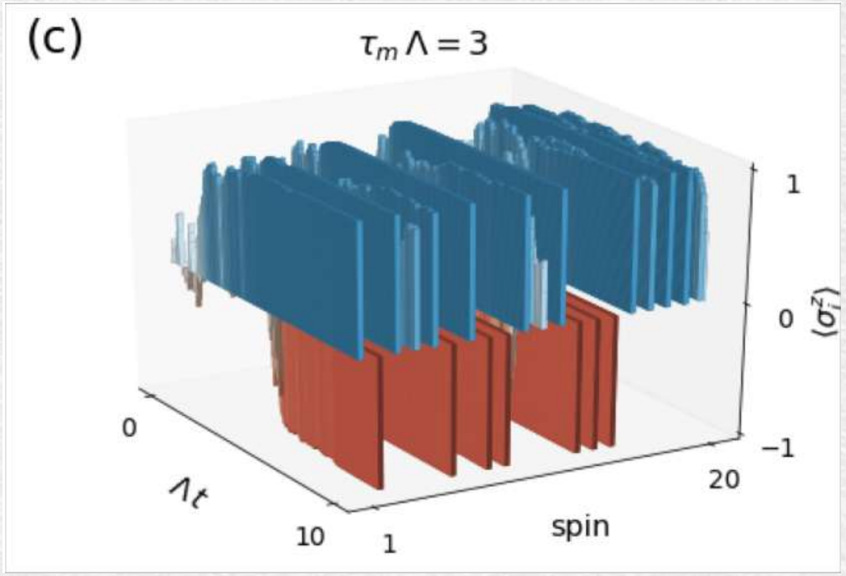


weak probing

intermediate



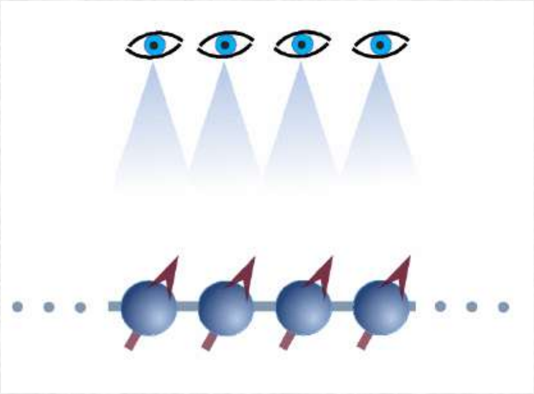
strong probing

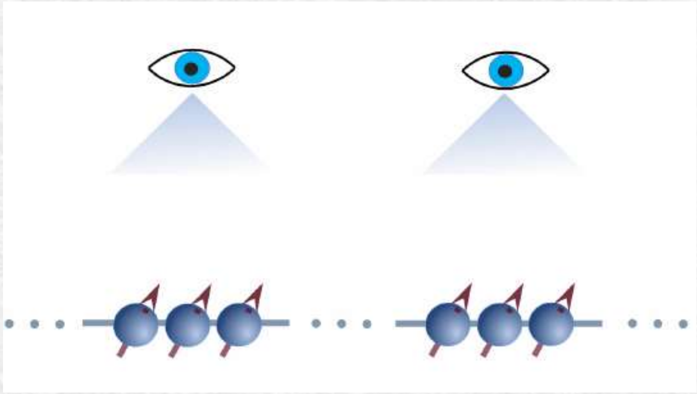


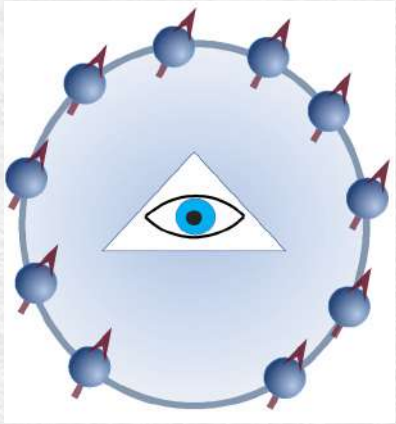
Garcia-Pintos, Tielas, del Campo, arXiv 2018

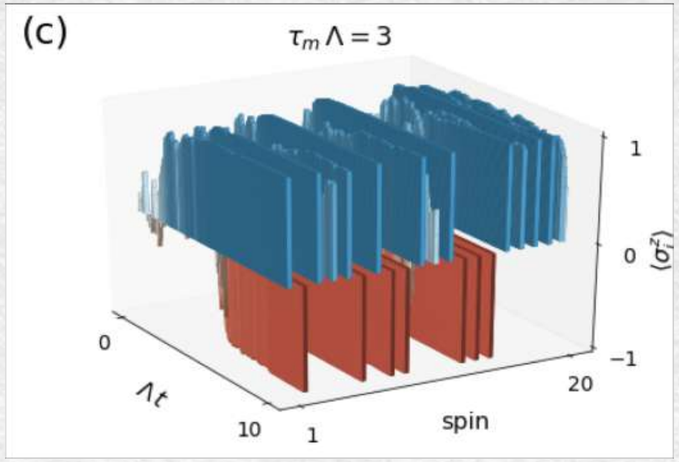
Symmetry breaking – effect of measurements

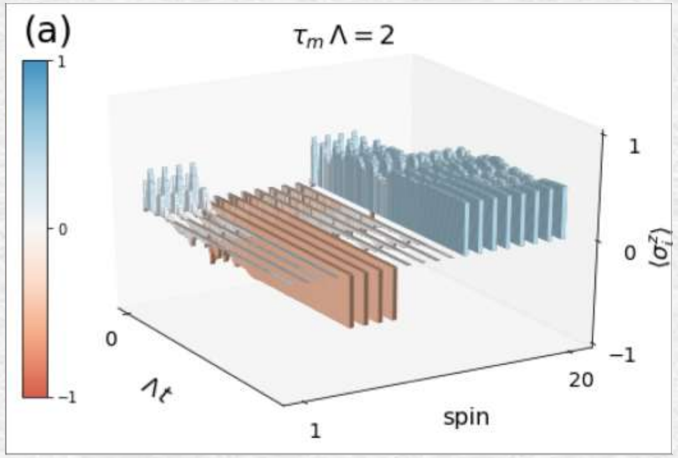
Coarse grained measurements

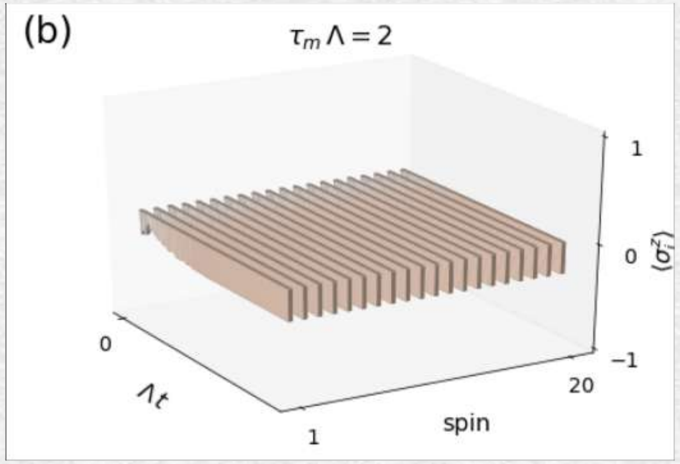








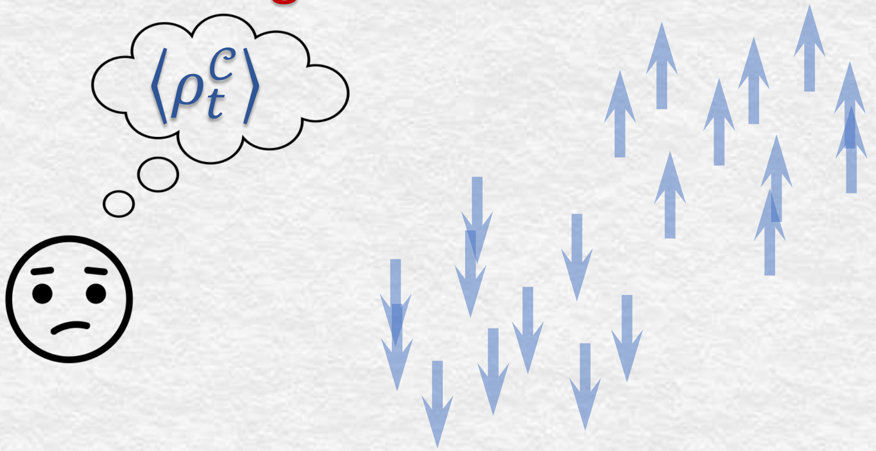




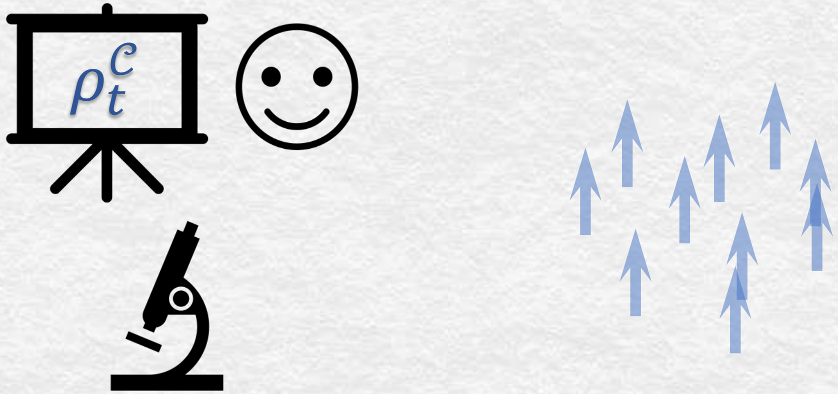
Garcia-Pintos, Tielas, del Campo, arXiv 2018

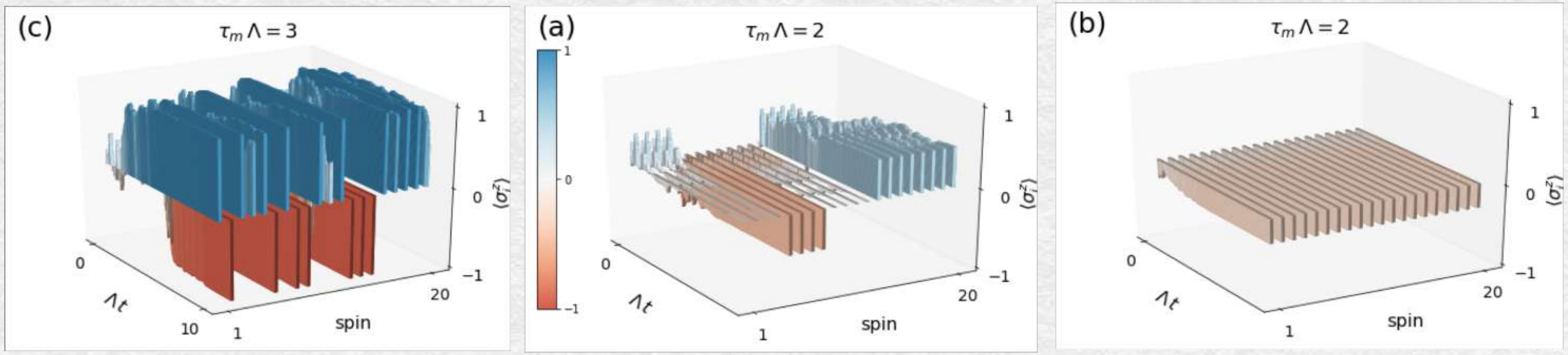
Symmetry breaking

evolution expected by ignorant agent does not distinguish states!



monitoring breaks symmetry in each realization





Monitoring agent can influence properties of symmetry-broken state

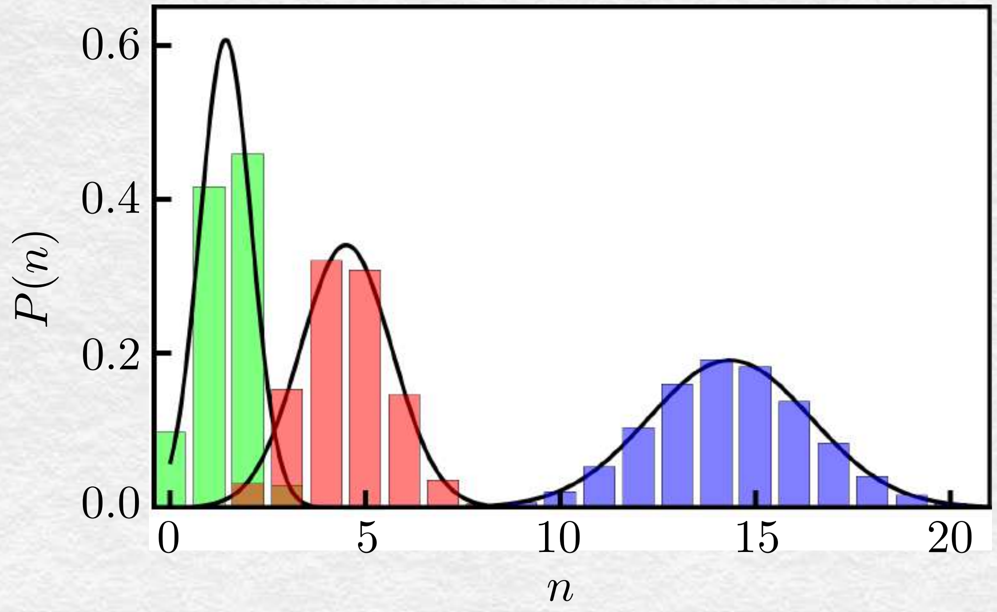
Garcia-Pintos, Tielas, del Campo, arXiv 2018

Outlook: Phase transitions in finite time: Defect suppression

SO FAR: ISOLATED SYSTEMS, NO MONITORING

What is known: Kibble-Zurek mechanism (KZM) predicts mean density of topological defects

News beyond KZM:
Full counting statistics of topological defects



The End

Thank you!

"Quantum speed limits under continuous quantum measurements",
Luis Pedro García-Pintos and Adolfo del Campo,
arXiv:1804.01600 (2018)

"Spontaneous symmetry breaking induced by quantum monitoring",
Luis Pedro García-Pintos, Diego Tielas, and Adolfo del Campo,
arXiv:1808.08343 (2018)

"Universal Statistics of Topological Defects Formed in a Quantum Phase Transition"
AdC, Phys. Rev. Lett. 121, 200601 (2018)